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THE ETHER IN A MAC CULLAGH GAUGE-TYPE THEORY

BY

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Abstract. The properties of ether in a Mac Cullagh gauge are analyzed. Precisely, we present the Maxwellian theory of light from a purely classical point of view, having natural ties with the constitutive laws of classical ether. As a consequence, the fundamental assumptions of the modern theory of nuclear matter can receive a natural philosophical status. In particular, we note that the properties of light are correlated with the properties of space, which is another way to say that we cannot see the space but through light. Moreover, our model allows us to analyze the properties of the electromagnetic field as a gauge field proper, giving us the unique opportunity of a sound natural philosophy along the classical lines.

Keywords: constitutive law; ether; Gauge Theory.

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1. Introduction

The Maxwellian theory of light presents it as a continuous space-time phenomenon. Only afterward has it been inferred that the electromagnetic theory of light should be a gauge theory. Here this theory is presented from a purely classical point of view, having natural ties with the constitutive laws of classical ether. It turns out to be very illustrative, mostly regarding the precise place and description of the gauge freedom introduced by the constitutive laws.

Many of the fundamental assumptions of the modern theory of nuclear matter can thus receive a natural philosophical status. In particular it is quite interesting that the properties of light are correlated with the properties of space, which is another way to say that we cannot see the space but through light.

2. Deformations and Tensions in a Natural Association

If we work under the only assumption that the stresses and deformations are to be represented by 3×3 matrices, then the constitutive equation connecting them can be taken as a natural starting point in the physical characterization of a continuum, even without assuming any potential function. As a matter of fact, this may be a good possibility to circumvent this unclear concept, because it might not even have a physical meaning for all the problems of continuum. The theory to be presented now seems to be of a great importance, not only concerning classical well known concepts, but mostly because it shows clearly what is the real difference between a gauge angle, so much in use today in the theory of skyrmions, and an angle proper as we know it from geometry. In hindsight it also connects Fresnel's theory of light with the modern Yang-Mills gauge theory, in a historical continuity through ones of the most important reference points of the electromagnetic theory of light.

Let us start by considering the theory of the constitutive laws (Mazilu and Agop, 2012; Agop and Mercheș, 2018) in its utmost generality, *i.e*. with no deformation potential, but only with the idea of distinctiveness of the matter in deformation. A constitutive law relating the stress and strain, must be *a priori* of the form

$$\mathbf{y} = \mathrm{p}_0\mathbf{e} + \mathrm{p}_1\mathbf{x} + \mathrm{p}_2\mathbf{x}^2 \tag{1}$$

where **e** is the unit 3×3 matrix. We can really call this equation a *natural* constitutive law, if not on the grounds that it appears frequently in Nature, at the very least due to the fact that it can be derived from natural considerations on our representation of stresses and strains. Indeed, if our models of stress and strain are 3×3 matrices, and if the constitutive law is to be analytic, the Eq. (1) must be automatically in effect. For, then, the relation between the two matrices can be represented by a formal series reducible to a second order polynomial by means of the Hamilton-Cayley theorem. By the very same token, that relation can be written just as well with the places of stress and strain matrices interchanged. Thus, strain as a function of stress is also a quadratic function, only with some other coefficients.

Now, a deeper insight in the problem of deformation in general shows a specific physical feature of it: one has to deal here with *uncontrollable manifestations*, which must be somehow reflected in the relation between stress and strain. This is quite obvious in the different attempts of descriptions of the plastic deformation of matter along the time (see, for instance, Hill, 1998, chapter one and, especially, two). In the particular case of nuclear matter, we can describe those uncontrollable manifestations starting from the observation that the nucleus *moves freely through the ether*. This is actually the essential characteristic of any kind of matter, if the ether exists: it moves freely through it. Insofar as it should be considered matter, the nucleus is therefore no exception, and this fact should be properly taken into consideration. Nevertheless, because the physical space occupied by nucleus is most certainly out of the reach of any of our senses, even assisted by technology as it were, we can hope, first that the assumption of continuity of nuclear matter is closer to truth there than in any other place, and secondly that the surface of the nucleus, regardless of its details, is the last frontier between matter and space.

There is a clear distinction between ether and matter here, not always taken properly into consideration, and the nuclear matter might help in clarifying the issue. Specifically, the ether is not matter, in the first place, so it shouldn't be treated by the theoretical means we use to treat the matter. Secondly, one can say that, by its very nature, the ether 'pervades' matter, which is another way to say that the matter has structures only in ether. Inasmuch as we are concerned with the ether as a category in itself, these very properties differentiate between two kinds of ether: *ether in space* and *ether in matter* (Larmor, 1900). It is in this last sense, of 'content' of matter, that we need to understand the above statement that 'ether pervades matter'. This notion will be made clear here, as we go on.

Extending this conclusion even to the limits where we cannot discern in any way a structure in matter, we can say that the matter is *made of ether*, but not vice versa: the ether is not a matter structure, as usually considered in classical physical analyses. Moreover, we can also say that the *ether in matter* and the *ether in space* are two species of the very same continuum, having however quite different physical and theoretical (mostly speculative) properties. And these properties are indeed reflected in the natural constitutive law above, originating from the representation of the stresses and strains. Let us see how.

In the constitutive law given by Eq. (1), the *material* it characterizes has a precise identity, given by the coefficients p_0, p_1, p_2. These should be accessible, in a certain way, to experiment. Finding these coefficients is what one actually means by 'material characterization' in the contemporary engineering practice. This practice consists of a set of standard experiments, specifically designed with the purpose to extract from them the material properties. In this particular case, the experiments should offer the coefficients p_0, p_1, p_2. Often times in the actual practice these coefficients are considered *pure material* properties, but this restriction confuses the issues, sometimes with serious consequences. Let us make this statement a little more explicit, in order to use it properly here.

No matter what the material properties are, the whole philosophy of their experimental origin hangs on the constitutive law, in our case the Eq. (1). Referring now the discussion to that equation, one can say that in each and every one of the experiments – the so-called loading experiments – with a material described by that equation, the principal directions of stress coincide with the principal directions of strain. On the other hand, if $y_{1,2,3}$ are the principal values (eigenvalues) of stress matrix, and $x_{1,2,3}$ are the principal values the strain matrix, according to the constitutive law (1) we must have satisfied the system of three equations with three unknowns – the material parameters:

$$y_k = p_0 + p_1 x_k + p_2 x_k^2; \quad k = 1,2,3 \tag{2}$$

Assume now that we are able to perform such experiments allowing us to *measureall three* principal values of strain and stress simultaneously. In practice, this is an impossible task, but let us assume it though, just for the sake of argument. The outcome of these experiments – the values $x_{1,2,3}$ and $y_{1,2,3}$ – will then allow us to algebraically calculate the material properties embodied in the coefficients $p_{0,1,2}$ from the system (2). This system has a nontrivial *unique* solution if, and only if, the determinant

$$\begin{vmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^2 \end{vmatrix} = (x_2 - x_3)(x_3 - x_1)(x_1 - x_2) \tag{3}$$

is non-null. Thus, the material parameters p_0, p_1, p_2 are uniquely determined, regardless of the character of imposed stress, by the solutions of the system (2) if, and only if, the resulting principal deformations *are all different from one another*.

However unique, and thus well suited for characterizing the material by its individuality in constitutive behavior, the coefficients thus obtained are by no means *pure material* properties, inasmuch as they depend on the *impressed state of stress*, and this one is accidental, to say the least. Therefore we are further required to state more precisely what we understand by *pure material* properties. This is, and indeed always was, an issue for the theory of constitutive laws. This issue cannot be addressed but by explicitly accepting the uncontrollability as an essential trait of the process of deformation.

Notice indeed that, in the practical science of materials *there are* deformations even in cases where there are no impressed stresses acting on our material, for instance in the relaxation and creep processes. In those cases something uncontrollable goes on inside matter, that changes its perceived properties, especially the shape. Inasmuch as we don't know their origin, because there is no apparent cause of them, these deformations are indeed some *intrinsic properties* of the material. They might be generated by forces of whose presence we have momentarily no idea, or else they can be true intrinsic properties that we still can model as stresses – the so-called internal stresses – having in fact the physical meaning of energy densities. Limiting, when it comes to description of ether, the mechanics only to *external* or *impressed* forces, leaves no alternative but to consider these energy densities as intrinsic properties of the continuum. In terms of the system (2) they can be described somehow by the system of equations:

$$\begin{aligned} 0 &= p_0 + p_1 x_1 + p_2 x_1^2 \\ 0 &= p_0 + p_1 x_2 + p_2 x_2^2 \\ 0 &= p_0 + p_1 x_3 + p_2 x_3^2 \end{aligned} \tag{4}$$

This system states that there are deformations of the material under no external stress. In this case, the material characterization by experiment is transferred into finding the solutions of this homogeneous linear system, in case they exist. Like in the regular engineering practice, the material properties are the coefficients $p_{0,1,2}$, measured in special conditions. These solutions always exist, we only have to decide just how many, and this depends on what we really can *always* measure. If we always measure three different deformations in three orthogonal directions in space, then this kind of matter is not responsive to the impressed stresses: all the material coefficients are necessarily zero. However, there are also possibilities of solutions in which the matter may be responsive to stresses, in other words its deformation is indeed accompanied by stresses. These are therefore characterized by nontrivial solutions of the system (3). Thus if we measure one and the same strain value in any direction in space, we have a double infinity of states of stress of such matter, depending on two material parameters. If we can measure two strain values, and only two, in a direction and its perpendicular plane for instance, then we have states of stress of the matter depending on one material parameter. Granting that we can include one of the material parameters into a measurable quantity, we can therefore nontrivially characterize a material exhibiting strain under no obvious stress. The most general constitutive law satisfied by such a material under a given general state of stress is

$$\mathbf{y} = K(\mathbf{x} - x_1\mathbf{e})(\mathbf{x} - x_2\mathbf{e}) \tag{5}$$

where K is an arbitrary constant having the dimensions of a stress, and **e** is the identity matrix. Such a material has *three* own uncontrollable characteristic quantities, of which only *two* are directly measurable.

In closing here, notice that as long as we are interested in just the measurable quantities, a convenient way to express a characteristic deformation matrix for a material exhibiting uncontrollable strains, is in the form of the tensor

$$x_{ij} = x_2\delta_{ij} + (x_1 - x_2)\cdot m_i m_j; \quad i, j = 1,2,3 \tag{6}$$

where $\hat{m}$ is a unit eigenvector, corresponding to the eigenvalue x_1. Such a material has distinguished directional properties with respect to the direction $\hat{m}$, and these properties are given by the eigenvalues x_1 and x_2. As a matter of fact, the Eq. (6) does contain both of the previous two cases of measurement as particulars, if we agree to characterize the *intrinsic material properties as deformations*. Notice that this is an assumption independent of the constitutive description and must be secured by our measurement capabilities in special cases. Thus we have this general conclusion: *whenever a material has the capability of deforming freely, i.e. under the action of no noticeable stresses, its deformation matrix must be of the form given by Eq. (6), all the particular cases included*. This is plainly the case of the ether of space, opposing no resistance to matter moving through it, where the deformations, as well as the stresses associated 'naturally' with them, are then manifestly tensors.

By the same token we can discuss that category of continuum capable of sustaining stresses and exhibit no strain. It is indeed by this essential property – the rigidity – that matter comes first to our senses. For this the converse constitutive law must be taken into consideration, namely

$$\mathbf{x} = q_0\mathbf{e} + q_1\mathbf{y} + q_2\mathbf{y}^2 \tag{7}$$

This time, however, **y** may be only abusively called stress; let us just say that it is a tensor representing the internal energy density in matter. Then the defining state of such a continuum will be characterized by the system of equations

$$\begin{aligned} 0 &= q_0 + q_1 y_1 + q_2 y_1^2 \\ 0 &= q_0 + q_1 y_2 + q_2 y_2^2 \\ 0 &= q_0 + q_1 y_3 + q_2 y_3^2 \end{aligned} \tag{8}$$

corresponding to no strain response. Again, the material characterization depends on the number of solutions of this system. And just like before the most general strain it exhibits is of the form

$$\mathbf{x} = K_1^{-1}(\mathbf{y} - y_1\mathbf{e})(\mathbf{y} - y_2\mathbf{e}) \tag{9}$$

where the constant K_1 has, again, dimensions of a stress. It is perhaps of some significance to recall that the relation (9) with $y_1 + y_2 = 0$ has been found by Bell (1968; 1973) to be, by and large, a characteristic for metals: they always struck our senses by their hardness. Besides, the Eq. (9) seems to come up with a specific 'quantization' discovered by Bell, which seems to be universal.

Again, as long as we are interested in just measurable quantities characterizing such a material, its distinctive stress tensor assumes the following convenient representation, similar to (6)

$$y_{ij} = y_2\delta_{ij} + (y_1 - y_2)\cdot n_i n_j; \quad i, j = 1,2,3 \tag{10}$$

where $\hat{n}$ is a unit vector corresponding to the eigenvalue y_1. One can say that the general characteristic of materials exhibiting no strain under stress is of the form (9), all the particular cases included.

Two points need to be clarified here. The first one regards the tensors from Eqs. (6) and (10). These equations are specific for matrices that we would like to term here as 'equivalent' to a vector field. We understand this equivalence in the following way: let $\vec{v}$ be a vector field, and let us construct the following matrix

$$v_{ij} = \alpha\delta_{ij} + \beta v_i v_j \tag{11}$$

It is clear that, because v_k are the components of a vector, and supposing that α and β are scalars, this gives v_{ij} as the components of a *tensor* indeed. One of the principal values of this tensor, namely α, is double. The other principal value, different from α, is given by

$$\alpha' = \alpha + \beta v^2 \tag{12}$$

Notice some interesting features of this kind of tensor. First of all, if either one of β and v_k is null, **v** is a purely spherical tensor. Secondly, if we calculate the eigenvector of **v**, corresponding to the eigenvalue (12), we find out that this eigenvector is $\vec{v}$, up to a normalization factor. This property is independent of the parameter α, and this is actually what we mean by the above-mentioned equivalence: given the vector $\vec{v}$ we can directly construct the tensor **v** as a family of two-parameter tensor matrices having it as an eigenvector. One can say that **v** represents a kind of action that points in the 'general direction' of $\vec{v}$, however not *exactly* in that direction. This is what we understand by 'equivalent to a vector field.

Secondly, we feel compelled to elaborate a little on the very concept of measurement here. Inasmuch as we ideally follow one or the other kind of materials described above, the measurements means measurement of either strains or stresses in the specific conditions in which Eqs. (4) or (8) are valid. Practically, however, this is not the case: for a real material those conditions are seldom satisfied. Besides the technological necessities push for immediate results, and we cannot wait for creep or relaxation results, which might take ages to get. Thus, practically, a measurement means in fact a curve-fit. We perform uniaxial experiments with a piece of material; this gives a privileged direction. Then we simply fit the results to a quadratic form, and from this we extract, for instance, the numbers K, x_1, x_2, from Eq. (5). Here we are entitled to say that only x_1 and x_2 are measurable because they can be referred to an ideal experimental setup.

3. The Classical Ether as a Deforming Medium

In order to convey what we think is the right meaning to some historical facts, we need to recall that the algebraic representation of a second order tensor is a quadratic form having the space form of a quadric (ellipsoid, hyperboloid or paraboloid). This was actually the classical way of description of light to Huygens and Fresnel (Huygens, 1690; Fresnel, 1827; Whittaker, 1910, Volume I, mainly chapter IV). The light was then seen as ether in extension. It is only afterwards that it was characterized as a perturbation propagating in space. Therefore it was important, for the genius of a geometer such as Huygens for instance, to characterize the space form of extension, *i.e.* the form of the wave as we say nowadays. It was a sphere or an ellipsoid of revolution, and these are quadratic forms associated with tensors like that given in Eq. (11). Indeed, in cases where the quadratic form associated to a second order tensor is positively defined, its space representation is always an ellipsoid whose semiaxes are given by the principal values of the tensor. It is clear that a spherical tensor – *i.e.* for $\beta = 0$ in Eq. (11) – represents a sphere, while the complete tensor (11) itself represents an ellipsoid of revolution (spheroid). This is the way Huygens characterized the propagation of light in vacuum, and the phenomenon of double refraction in transparent matter. In our terms here one might say indeed that Huygens just noticed that the ether entering the structure of matter is characterized by a tensor like (11). One may say further that, in depicting the double refraction, Huygens considered both cases, β zero and nonzero, *separately*, thus theoretically accounting for this strange phenomenon. It was the merit of Fresnel to notice that a single space form as related to the general tensor (11) is quite sufficient in order to characterize the double refraction. In doing this he just noticed the important fact, taken for granted nowadays, that the thing we are after in such a construction is actually the eigenvalue of the tensor representing the ellipsoid, because it is in relation with the *speed of light*. And, according to the classical principles, it is the speed of light that changes in the phenomenon of refraction. Only, it has later been noticed, this change is not done according to the rules of classical dynamics, which are mainly vector rules.

Speaking of this moment in Fresnel's thinking, and ours for that matter, let us notice here a fact that we find to be of special importance for what has followed afterwards, especially for the prototype of the nonabelian gauge theories, the Yang-Mills theory. Namely replacing the Huygens' double construction – sphere and spheroid – by a single general construction – spheroid – carries over into space forms the mark of a certain property of linearity of the tensors from the family given by Eq. (11): the linear combination of any number of tensors 'belonging' to a vector is always a tensor of the same family, *i.e*. it belongs to the same vector. It is indeed this property that allowed Fresnel to see that in the general case, of crystals having two axes of double refraction, the right description is that by a quadratic form representing not a spheroid but a general ellipsoid (Whittaker, 1910). Relegating the reader to historical works already indicated, we try here a more limited task, namely to answer the question: what would have happened if Fresnel would continue his logic, based however upon the existence of *two* kinds of tensors (11) one related to the *ether in space* the other related to *ether in matter*?

This question comes out of the simple observation that the ether in space does not oppose any resistance to the motion of matter, *i.e*. it has no stresses under the obviously nontrivial deformation induced by the motion of the bodies. On the other hand, while in matter, the ether has the property of impenetrability, *i.e*. it exhibits nontrivial stresses under no deformation. Within the limits of the theory presented above, this philosophy could have been materialized even from the times of Fresnel, by the proposal of James Mac Cullagh (Mac Cullagh, 1831) for the representation of the light phenomenon according to Newtonian view of forces. Let us briefly see what Mac Cullagh's philosophy is about.

Mac Cullagh was concerned with the elliptically polarized light, like the light passing through rock crystals. He found that this can be represented by two harmonic vector processes in the same plane, like the ones invented by Fresnel, making a certain angle between them. Later on (Mac Cullagh, 1836) he noticed that the theory can be put in a space-time form by a system of coupled differential equations, which led him to the foundations of the theory of ether (Mac Cullagh, 1839) – later improved by Lord Kelvin and Larmor – and finally to an exquisite explanation of the phenomenon of double refraction in quartz (Mac Cullagh, 1840). It is to be noticed that the veiled compelling argument of Mac Cullagh seems to have been the faulty notion of displacement to Fresnel. Indeed, in the case of light – a continuum phenomenon – the mechanical displacement has no object, *i.e*. it is not referring to a material point, but simply to a position in space, apparently without matter, as we know it, located there. This very fact made Newton's natural philosophy hardly relevant to the light, a detail corrected in a brilliant way by Mac Cullagh. These facts explain, by and large, the almost explicit contribution of Mac Cullagh to the future electromagnetic theory of light (Darrigol, 2002; Darrigol, 2010). In hindsight though, Mac Cullagh's seems to us to be more than an electromagnetic theory. It is indeed the very first specimen of a gauge theory, of the kind that came into existence more than a century afterwards in the form of the Yang-Mills theory (Yang and Mills, 1954).

4. Witten's Ansatz: Instantons from Skyrmions or Vice Versa

The simplest way to get the characterization of ether inside a biaxially refringent crystal for instance, is by letting enter the play the fact that *there exists ether in space and also ether in matter*. And we do this in a special manner, which we venture to call *Witten ansatz*, because it has intimate ties with the known gauge solutions put forward by this theoretician (Witten, 1977). Indeed, Witten's original work addresses the problem of possibility of constructing general solutions for the Yang-Mills gauge fields – the so-called instantons. These gauge fields appear to be, by what we just have shown here, and we'll continue to elaborate on, a natural generalization of the classical Maxwell stress theory to a connection of the space. Indeed, as long as we are talking of stress and strain, we are talking a fortiori of skyrmions first, and therefore, before talking of "skyrmions from instantons", as it is nowadays the theoretical habit ever since the works of Atiyah and Manton (Atyiah and Manton, 1989; Atyiah and Manton,1993), we need first to talk of "instantons from skyrmions", as it was historically the case.

Here a Witten-type ansatz can be easily obtained for the fields only, by admitting that the ether is characterized not by a single tensor of the general type (11), but by two, therefore by two characteristic vectors, $\vec{u}$ and $\vec{v}$ say – like Mac Cullagh's vectors for instance. According to the logic just outlined above, the complete tensor describing the ether in general would then be, at least in a first instance, a linear combination of the two, something of the form:

$$w_{ij} = \alpha\delta_{ij} + \beta u_i u_j + \gamma v_i v_j \tag{13}$$

With the classical electromagnetic theory in mind, we may need to notice that the calculations are more symmetrical if we write (13) in a more convenient form as

$$w_{ij} = \lambda u_{ij} + \mu v_{ij} \tag{14}$$

where λ and μ are two real parameters, describing the degree of 'space or matter' in the 'constitution' of ether per se. Here the matrices **u** and **v** are defined by

$$\begin{gathered} u_{ij} = u_i u_j - \frac{1}{2}u^2\delta_{ij}\,; v_{ij} = v_i v_j - \frac{1}{2}v^2\delta_{ij} \\ u^2 = u_1^2 + u_2^2 + u_3^2; \quad v^2 = v_1^2 + v_2^2 + v_3^2 \end{gathered} \tag{15}$$

This tensor contains eight measurable quantities: λ, μ, and the two 'intrinsic' vectors. It represents a 'linear structure', so to speak, of the ether. Indeed, as a category apart, according to Larmor's idea, the ether can be said to exist in space as well as in matter, and this is reflected in Eq. (13). The form (14) of the tensor **w** is only a convenient way to express the same idea, whereby the tensors **u** and **v** characterize the two subcategories of ether.

Written at length, the tensor (14) is

$$w_{ij} = \lambda u_i u_j + \mu v_i v_j - \frac{1}{2}(\lambda u^2 + \mu v^2)\delta_{ij} \tag{16}$$

It is easy to see that it has three real eigenvalues. Indeed, its orthogonal invariants are

$$I_1 = -e; I_2 = -e^2 + g^2; I_3 = -e(e^2 - g^2) \tag{17}$$

where we denoted

$$e \equiv \frac{1}{2}(\lambda u^2 + \mu v^2); \quad \vec{g} \equiv \sqrt{\lambda\mu}\,(\vec{u} \times \vec{v}) \tag{18}$$

The eigenvalues of **w** can then be calculated as the roots of the corresponding characteristic equation, and they are

$$w_1 = e, \qquad w_{2,3} = \pm\sqrt{e^2 - g^2} \tag{19}$$

It turns out that the pair from Eq. (18) gives one eigenvector of **w** and the corresponding eigenvalue. The other two eigenvectors of **w** are orthogonal, and located in the plane of the vectors $\vec{u}$ and $\vec{v}$.

We can read on Eqs. (16) and (18) the classical form of the Maxwell stress tensor of an electromagnetic field in free space. Therefore the previous, 'neo-fresnelian' as it were, Witten-type characterization of the ether actually coincides with the electromagnetic description of the light, which took shape some half-century after Fresnel and Mac Cullagh. However it tells us much more than the electromagnetism per se, as indeed the case should be. First of all, we can appreciate the fact that a characterization of the ether, as it comes out of light measurements, is a highly idealized situation, if we think it in terms of the electromagnetic theory. Indeed, this last theory is naturally connected with the idea of vector representing the electromagnetic field, deriving from the very same property of classical force. What one can actually measure in a point in space is an average of the influences of ether and matter, and it is this fact that has to be taken into consideration when talking, for instance, of the *cosmic background radiation*, or in any bolometric measurements for that matter. Thus, we don't measure averages of some vector fields, as the electromagnetic theory of light almost tacitly assumes, but rather averages related to a tensor field. This may largely explain why the Planck spectrum lets itself be exquisitely described by a Gaussian probability density of the frequency at a given temperature (Priest, 1919a; Priest, 1919b).

Now, as long as we represent the structure of ether by a tensor, this one has two kinds of space averages attached to it in any space point: one of them is the *average normal component* the other is the *average tangential component* of the tensor in that point (Mazilu and Agop, 2012). Indeed, for every *plane in space* through a certain point, a tensor has two characteristic scalar intensities associated with it: the normal and in-plane (tangential, or shear) intensities. What we want to say is that the description of light involves statistics in the first place, and we cannot discuss the light experimentally but only based on statistical estimators: the experiment cannot give anything else. On the other hand, it is this very aspect of the physics of light that gave us the possibility to describe it electromagnetically. Nevertheless, one can see that, no matter how attractive when it comes to describing the behavior of light, this is only a particular case, involving experiments where the ‘property of linearity’ in the behavior of ether is conspicuous.

One of the most important consequences of this view is the fact that the characteristics of an electromagnetic field in vacuum are statistical, not geometrical properties, which is why they *should be considered gauge fields* in the first place. Historically, this was a subject of great debate that led to the actual science of ellipsometry, and further to the theory of coherence of light. In order to show its deep significance, let us explain here how a known geometrical property of electromagnetic field, namely the perpendicularity of electric and magnetic components of an electromagnetic wave in vacuum, comes out as a ‘statistical’ property related to a gauge. It is worth, indeed, considering this issue a little closer, because it shows how a universal Huygens’ principle, *i.e.* a Huygens principle according to Fresnelian view of the light, can be connected with the measurement of the electromagnetic field quantities.

Consider, therefore, the tensor **w**, whose eigenvalues are given in Eqs. (19). Assume a local reference frame given by its eigenvectors. Then the corresponding vector associated to this matrix is

$$\left|w\right\rangle \equiv \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} \tag{20}$$

With Eqs. (19) the component of this vector perpendicular to the octahedral plane (Mazilu and Agop, 2012) is given by

$$w_n \equiv \left\langle w \middle| n \right\rangle = -\frac{2}{\sqrt{3}}e \tag{21}$$

On the other hand, the component of the vector (20) in the octahedral plane (shear component) is given by

$$\left|w_t\right\rangle = \frac{2}{3}\begin{pmatrix} -2e \\ 3\sqrt{e^2-\vec{g}^2}+e \\ -3\sqrt{e^2-\vec{g}^2}+e \end{pmatrix} \tag{22}$$

Therefore the magnitude of this shear vector is

$$\left\langle w_t \middle| w_t \right\rangle = \frac{8}{3}(4e^2-3\vec{g}^2) \tag{23}$$

Now, in order to calculate the orientation of the vector (22) in the octahedral plane, we need to choose a reference direction in that plane.

This choice brings us again over to the theory of deformations, the one that stays at the basis of Manton geometrization (Manton, 1987). Indeed, disregarding translations and rotations, a deformation maintaining the property of *constant curvature* of a three-dimensional Riemann space is formally given by a metric tensor like that from Eq. (11) above (Coll *et al*., 2002). The main property of this tensor, is that it closely represents our practice with deformations: the deformation tensor considered as a variation of the metric tensor, is always quadratic in the components of a vector which can deservedly be called stretch. This is, indeed, an essential characteristic of the classical theory of deformations, when they are theoretically explained as gradients and experimentally described by stretches. Then, for such a tensor we have (Mazilu and Agop, 2012)

$$\left\langle h \middle| n \right\rangle = f + \frac{g}{3}\vec{\lambda}^2, \quad \left|h_t\right\rangle = \frac{2}{3}g\vec{\lambda}^2\begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} \tag{24}$$

If the stretch vector $\vec{\lambda}$ is perpendicular on both $\vec{u}$ and $\vec{v}$, *i.e.* it is in the direction of the eigenvector from Eq. (18), then the tensors **w** and **h** commute. Thus they have a *common reference frame* in any point, given by their eigenvectors, and so it can be arranged that their octahedral planes coincide. In this case the direction of the shear vector from Eq. (24), *which is fixed*, can be naturally chosen as a reference direction in the octahedral plane. Then the angle ϕ of the vector (22) with respect to this fixed direction, is given by

$$\cos\phi = -\frac{e}{\sqrt{4e^2-3\vec{g}^2}} \tag{25}$$

This shows that, under the specified conditions, *i.e*. for deformation at constant curvature, the angle ϕ is independent of the reference stretch vector $\vec{\lambda}$, and depends only on the properties of the ether, as represented by the two vector fields $\vec{u}$ and $\vec{v}$. This angle is therefore what we would like to call a *gauge* *angle*. With a proper choice of sign for the square root, its origin, $\phi = 0$, occurs for e = g. This condition means, in turn, that the angle θ between the vectors $\vec{u}$ and $\vec{v}$ – a proper geometric angle this time, for it is the angle of two directions in space – is given by equation

$$|\sin\theta| = \frac{1}{2}\left|\frac{\lambda u^2 + \mu v^2}{\sqrt{\lambda\mu}\,uv}\right| \qquad (26)$$

As the quantity from the right hand side here is always greater than or equal to 1, the angle between vectors $\vec{u}$ and $\vec{v}$ cannot be but 90°. But this is the well known property of the electromagnetic theory of light in vacuum.

5. Conclusions

Let us therefore conclude that Fresnel's expression for Huygens' principle, as obtained from a Witten-type ansatz referring to two Mac Cullagh vectors, requires a certain characteristic of the tensor **w** representing the ether. Specifically, the gauge angle giving the orientation of the shear vector of ether in the local octahedral plane is given strictly by the vector fields entering the structure of tensor **w**. Now, it is to be naturally assumed that in the conditions of vacuum, the space deformation must be 'aligned', so to speak, with the tensor **w**, in the manner needed in order to have a common local reference frame. They can be 'misaligned' only in matter. In other words the matter makes its presence noticed by this very misalignment, which should be somehow reflected in the values of the gauge angle different from 0°. Therefore only the ether in vacuum is characterized by Eq. (26) and thus by a 90° geometrical angle between the vectors $\vec{u}$ and $\vec{v}$. This is a general characteristic of the electromagnetic fields in vacuum which, as shown here, is not a consequence of their vector character, but of the tensor character of the physical quantities involved in the measurement process. One can surely say, therefore, that it is a statistical property, and that inside matter it should be apparent in a specific way, somehow related though to the considerations above.

At this point we have to stop a little, in order to offer one more argument to our philosophy of geometrical representation of nuclear matter. Certainly the Eq. (26) does not allow but only the value 1 for the sine of the geometrical angle between the two vectors. Therefore any value, other than zero of the gauge angle ϕ is to be prohibited on this basis. One can, of course, argue that the electromagnetic theory is quite a particular approach for the problem of ether, and indeed this is the very case here: the ether we are talking about is that of vacuum. It is in matter that the gauge angle has to be nonzero, as we mentioned above, *i.e.* only in matter the value of the ratio from the right hand side of Eq. (25) is different from unity. In that case e has to be different from g, and as the quantity from the right hand side of Eq. (26) is always greater than one in absolute value, the geometrical angle θ between the two vectors $\vec{u}$ and $\vec{v}$ has necessarily to be a complex angle. This is to say that the very geometry of the electromagnetic fields in matter is no more the usual Euclidean geometry, but rather a hyperbolic geometry for instance. Whence, once again, the argument that the hyperbolic geometry characterizes the nuclear space in the classical Kepler problem. This time, however, it comes out from the properties of the electromagnetic field as a gauge field proper. This should be the case of the gamma radiation of the nuclei, for example. Allowing, therefore, to Skyrme theory, the full freedom offered by Manton's geometrization, gives us the unique opportunity of a sound natural philosophy along the classical lines.

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ETERUL ÎN TEORII DE ETALONARE DE TIP MAC CULLAGH

(Rezumat)

În prezenta lucrare se analizează proprietățile eterului într-o teorie de etalonare de tip Mac Cullagh. Mai precis, prezentăm teoria Maxwelliană a luminii din punct de vedere clasic, având conexiuni naturale cu legile constitutive ale eterului clasic. În consecință, ipotezelor fundamentale asupra teoriilor moderne ale materiei nucleare li se poate atribui un statut ca cel dat prin principiile matematice ale filozofiei natural dezvoltate de Newton. În particular, proprietățile luminii pot fi corelate cu proprietățile spațiului, altfel spus nu putem „percepe” spațiul decât prin lumină. Mai mult, modelul nostru ne permite să analizăm proprietățile câmpului electromagnetic ca un câmp natural de etalonare, oferindu-ne oportunitatea unică a unei filozofii natural argumentată în sens clasic.